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# Theory of disordered contacts in high magnetic fields: strong disorder

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**Abstract.** We present an analysis of the behaviour of a strongly disordered lead in a magnetic field connected to a less disordered sample. The disordered lead acts as an imperfect contact, coupled in varying degrees to different outgoing and incoming states in the sample: in a magnetic field these states are the different edge states. For strong disorder what would be the inner edge state in the absence of disorder does not propagate through the disordered lead, and we discuss what happens in this case. We show how an activated occupation of the inner edge state results from the different energies at which the inner Landau level starts to propagate for different disorders. We present numerical results for some particular disordered leads, quantifying the difference in energies.

## 1. Introduction

In the preceding paper, we discussed, in the context of a model of weak scattering between edge states, how a disordered wire populates in different proportions the edge states in a clean wire to which it is connected, and how this explains the experimental results of Geim *et al* [1]. In this paper we consider the regime of strong disorder.

In the experiment of Geim *et al* [1] the sample was in essence a clean wire with the same parameters, except for the disorder, as the lead. We are particularly interested in the behaviour of the disordered lead in the regime where there is backscattering in the relatively clean wire, and we discuss below how this implies that the inner edge state of the clean wire is strongly backscattered by the disordered wire.

We have performed calculations of the conductance of disordered leads numerically. At zero temperature the transition between quantum Hall states is shifted to higher energies in the disordered lead; for a Fermi energy below the transition this leads to an activated occupation of the outer edge state in the sample. Close to the cut-off of the conduction of the inner mode through the disordered lead, the temperature dependence of the longitudinal conductance in the disordered wire dominates, and gives an occupation of the inner edge state varying as $\exp(-\alpha T^{-1/2})$.

## 2. Activated behaviour

Geim *et al* [1] have recently performed experiments on a Hall bar with disordered leads, as sketched in figure 1. They have observed the Shubnikov–de Haas oscillations arising from

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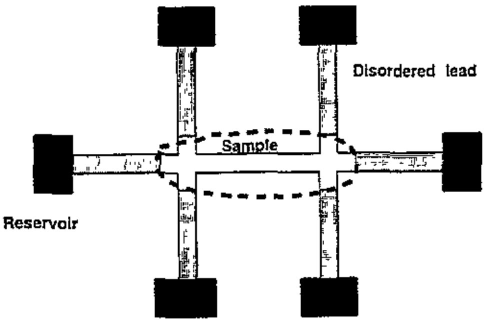


**Figure 1.** The experiment of Geim *et al* [1]. A Hall bar is connected to disordered leads (shaded) which are connected at one end to ideal reservoirs (black boxes), and at the other end to the region we refer to as the sample, which is enclosed in a dotted line.

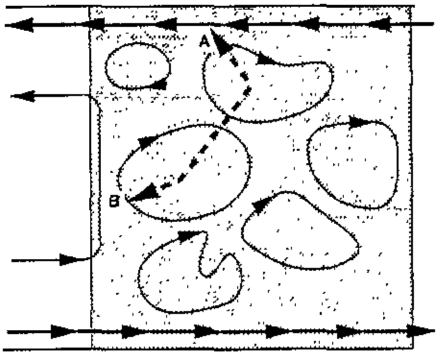


**Figure 2.** Strong-backscattering limit where electrons have to travel though the bulk of the wire, for example from A to B, to move from the inner edge state in the clean lead to the outer edge state in the disordered section. The dotted line shows a possible path that the current between A and B might take, hopping between the loops of circulating current.

the resonant backscattering of electrons in the central part of the bar [2], and have found that these oscillations are suppressed at low temperatures. We understand the suppression to result from a non-equilibrium occupation of the edge states in the sample by the disordered leads. Specifically, at lower temperatures fewer electrons are injected from the disordered leads into the inner edge state in the sample. There are therefore fewer electrons to be backscattered, and so the oscillations are suppressed. Away from the maximum of the Shubnikov–de Haas oscillations, a resistance proportional to $\exp(-\alpha T^{-1/2})$ was observed. Around the maximum, a resistance proportional to $\exp(-\alpha T^{-1})$ was found.

To understand the activated behaviour, we think directly about the occupation of the outgoing outer edge state as a function of energy and disorder. Let us take some fixed magnetic field and disorder. As the energy is increased in a wire, successive edge states will become able to propagate through the wire essentially without backscattering: the higher the disorder, the higher the energy necessary. Close to the point where an edge state is being closed off in the clean wire, that mode cannot propagate far in the disordered lead. At higher energies that mode would propagate far into the lead, so that at higher temperatures some fraction of electrons from the tail of the Fermi–Dirac distribution can propagate into the lead. The number of electrons emerging from the disordered lead into the clean wire within the inner edge state is then expected to be proportional to $\exp(-(E_0 - E_F)/T)$ where $E_0$ is the energy where the outer edge state starts to propagate in the wire, and $E_F$ is the Fermi energy. ($E_0 > E_F$.) We expect $E_0$ to vary as

$$E_0 = \hbar\omega_c(n + \tfrac{1}{2}) + E_D \qquad (1)$$

where $\hbar\omega_c(n+\frac{1}{2})$ is the energy at which the corresponding Landau level starts to be occupied in the bulk, and $E_D$ is a magnetic-field-independent offset. We therefore predict an activation energy increasing linearly with the magnetic field, as at higher magnetic fields increasingly higher energies are needed for the edge state to propagate in the disordered region.

Where the model of edge states propagating through the disordered region breaks down, a picture like figure 2 will be appropriate, where the inner edge state from the clean region is almost entirely reflected from the disordered sample, and exists only in localized rings in the disordered region. Figure 2 can be compared with figure 1 in the preceding paper [3] where in the weak-disorder limit [3] the system can be described in terms of backscattering between the inner edge states travelling in opposite directions, and interedge scattering between the inner and outer edge states on the one side. In this strong-disorder limit, all population of the inner edge state occurs by transfer through the bulk, that is the area of the disordered lead apart from the propagating outer edge states, and there is no helpful distinction between backscattering and interedge state scattering. The probability of an electron being transferred from the inner edge state to one of the edge states propagating in the disordered region, or vice versa, is then, for weak conduction, proportional to the conductance of the bulk region [4]. As the temperature increases, and so the conductance of the bulk region increases, the coupling to the inner edge state in the sample increases. If this bulk conductance is proportional to $\exp(\alpha T^{-1/2})$, consistent with experiments on the quantum Hall effect [5, 6] and theory [7, 8, 9], then the coupling of the lead to the inner edge state in the sample is also proportional to $\exp(\alpha T^{-1/2})$, which is the experimental temperature dependence away from the maximum of the Shubnikov–de Haas oscillations.

## 3. Numerical modelling of edge state distribution

We have argued that an activated occupation of the inner edge state occurs near the transition between different numbers of propagating edge states. We have performed numerical calculations to put a scale to this activation energy.

We have calculated the ballistic, that is without inelastic scattering, electron transport through a disordered lead for varying disorder, width and length. Standard recursive Green function methods [10, 11] were used to calculate the conductance of the system from the two-terminal Landauer formula [12, 13]. We used a simple estimate for the disorder, assuming linear, Thomas–Fermi screening in the 2DEG and uncorrelated ionized donors to calculate the potential. We neglected non-linear screening effects [14] since we are not interested in the region where there are few electrons in the lead, but rather the region where a higher mode is switching on. A full calculation would take into account the different screening, depending self-consistently on the occupation of the Landau levels in the wire. The use of uncorrelated ionized donors overestimates the disorder [15, 16, 17]. A rough estimate of the effect of correlations on the disorder is given by assuming that the electrons left in the donor region screen the potential there in a Thomas–Fermi manner. This suggests that the disorder in a typical 2DEG, as measured by the standard deviation of the potential, should be of the order of ten times smaller than would be found with uncorrelated donors. For the calculations we use the potential derived from randomly positioned ionized donors multiplied by a scaling factor. In Geim's experiment, we estimate that since the mobility in the disordered region was of the order of 100 times lower than in the clean wires, the disorder was of order 10 times as large as in the clean wires.

Since the correlations in the donors and the deliberately increased disorder in Geim's wires approximately cancel each other, we have taken the scaling factor to be unity, and we have used in this paper the parameters used by Nixon and Davies [14]. Namely we assume a donor to 2DEG separation of 28 nm, and assume an equipotential backgate at 70 nm from the 2DEG. We assume $2 \times 10^{16}$ m$^{-2}$ randomly positioned, ionized donors.

Figure 3 shows representative results for the conductance of such a disordered lead in the region where an edge state is closing off as the energy is lowered. The results show the shift of the transition to higher energies (or at fixed energy, lower magnetic field) when disorder is present. Calculations for different magnitudes of the disorder show a shift

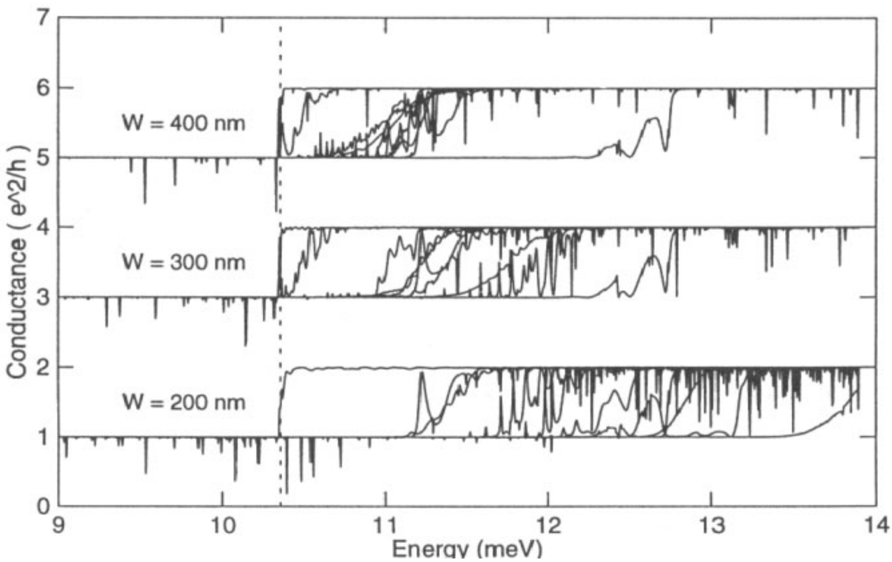


**Figure 3.** Conductance of 120 nm long disordered wires of different widths, $W$, for 10 random instances of the disorder. The parameters of the disorder are described in the text. The dotted line shows where the inner edge state would disappear in a perfect wire.

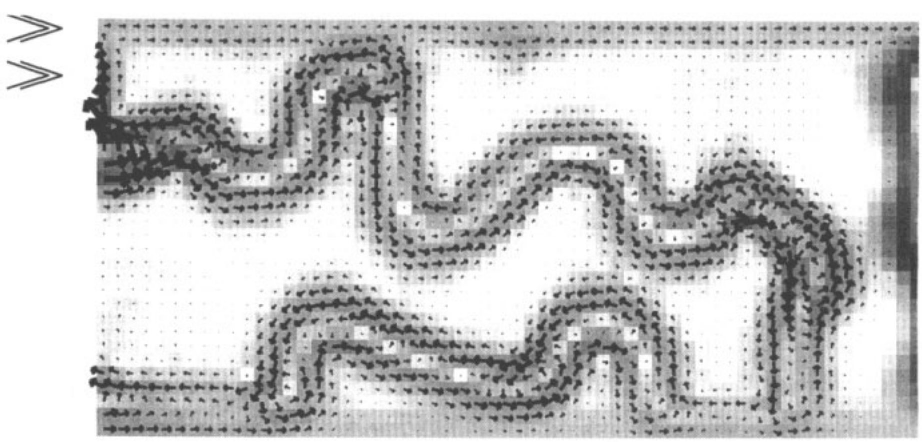

**Figure 4.** Reflected edge state in the energy range where the inner edge state would propagate in a clean wire, but does not propagate along the disordered wire. The figure shows the disordered section of the wire, which is connected to clean wires at both ends. The double chevrons mark where the edge states are injected. The shading marks the local density of states, the darker areas indicate the higher density.

essentially proportional to the disorder. Note the wide variation in the size of the shift, and in the width of the transition, from sample to sample.

The results are consistent with the disordered lead behaving as if it had a barrier at some height. Figure 4 shows a representative current pattern from the regime where the outer edge state is not propagating in the disordered region, but will still propagate in a clean lead. The inner edge state is reflected by the potential, in this case at the far end of the disordered region from where it was injected. We find that the shift in energy is generally proportional to the size of the disorder, consistent with a semi-classical picture where edge states are following equipotentials and the barrier height is determined by the highest equipotential crossing the sample.

The size of the shift in the transition depends on the size of the sample. In an infinitely wide wire this would be the single, percolating, equipotential at the mean value of the potential. In a very thin sample the barrier height is given simply by the maximum of the potential. The shift in the energy where the mode starts to propagate therefore is typically smaller for wider wires. As the wire becomes longer the shift will typically increase as higher equipotentials are included. A more complete theory, which we do not attempt here, would consider the extra scattering between the edge states, and equilibration within the edge states at finite temperatures, which will set a limit to the effective length of the wire.

## 4. Conclusions

We have presented a discussion of edge states propagating in a strongly disordered lead. For energies where the inner edge state propagates in a clean wire, but only propagates for a short distance in the disordered leads, there is only a weak coupling between electrons in the inner edge state of the clean wire and the disordered lead. We discussed two aspects of how the electrons are coupled to the disordered lead: firstly, at higher energies the inner edge state propagates through the disordered lead, giving an activated contribution to the coupling; secondly, electrons can be transmitted via the bulk part of the disordered lead where circulating edge states are expected. In a semiclassical picture, the energy at which an edge state starts to propagate through the disordered wire increases linearly with the disorder, and with the magnetic field. This energy is seen to vary strongly from sample to sample.

In this paper and the preceding paper [3] we have considered two models of a disordered wire connected as a lead to a relatively non-disordered sample, and discussed how they relate to the experiment of Geim *et al* [1]. In the first paper we considered a model of weak scattering between edge states. Whilst this helps to understand the qualitative trends in the experimental results with disorder and geometry, the weak-scattering model was unable to explain plausibly the dependence on the temperature. In this paper we have considered what happens in the limit of strong scattering. We have argued qualitatively how the two observed behaviours as a function of temperature can be explained, and we have presented numerical results showing the effect of plausible parameters for the disorder.

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## References

[1] Geim A K, Main P C, Taboryski R, Carmona H, Brown C V, Foster T J, Oakeshott R B S and Veje E 1994 *Solid-State Electron.* **37** 1005–9

[2] Geim A K, Main P C, Brown C V, Carmona H, Foster T J, Taboryski R and Lindelof P E 1994 *Proc. EP2DS10* at press
[3] Oakeshott R B S 1994 *J. Phys. Condens. Matter* **23** 4317
[4] In a previous paper [3] we showed that for weak disorder the probability of an electron being absorbed varied as the square root of the inter edge scattering rate. The geometry in the strong-disorder limit is different since we assume that all electrons reaching the edge of the sample are thermalized, and that the electrons are able to hop between many sites in the bulk of the wire.
[5] Ebert G, von Klitzing K, Probst C, Schuberth E, Ploog K and Weimann G 1983 *Solid State Commun.* **45** 625
[6] Briggs A, Guldner Y, Vieran J P, Voos M, Hirtz J P and Razeghi M 1983 *Phys. Rev.* B **27** 6549
[7] Ono Y 1982 *J. Phys. Soc. Japan* **51** 237
[8] Polyakov D G and Shklovskii B I 1993 *Phys. Rev. Lett.* **70** 3796
[9] Zhao H L and Feng S 1993 *Phys. Rev. Lett.* **70** 4134
[10] Sols F, Macucci M, Ravaioli U and Hess K 1989 *J. Appl. Phys.* **66** 3892
[11] MacKinnon A 1985 *Z. Phys.* B **59** 385
[12] Landauer R 1989 *J. Phys.: Condens. Matter* **1** 8099
[13] Büttiker M B, Imry Y, Landauer R and Pinhas S 1985 *Phys. Rev.* B **31** 6207
[14] Nixon J A and Davies J H 1990 *Phys. Rev.* B **41** 7929
[15] Efros A L, Pikus F G and Samsonidze G G 1990 *Phys. Rev.* B **41** 8295
[16] Kossut J, Wilamowski Z, Dietl T and Świątek K 1991 *Acta Phys. Pol.* A **79** 49
[17] Monroe D 1991 *Appl. Phys. Lett.* **59** 2293